NAG Toolbox for MATLAB

s17dg

1 Purpose

s17dg returns the value of the Airy function $\mathrm{Ai}(z)$ or its derivative $\mathrm{Ai}'(z)$ for complex z, with an option for exponential scaling.

2 Syntax

```
[ai, nz, ifail] = s17dg(deriv, z, scal)
```

3 Description

s17dg returns a value for the Airy function $\mathrm{Ai}(z)$ or its derivative $\mathrm{Ai}'(z)$, where z is complex, $-\pi < \arg z \leq \pi$. Optionally, the value is scaled by the factor $e^{2z\sqrt{z}/3}$.

The function is derived from the function CAIRY in Amos 1986. It is based on the relations $\mathrm{Ai}(z) = \dfrac{\sqrt{z}K_{1/3}(w)}{\pi\sqrt{3}}$, and $\mathrm{Ai}'(z) = \dfrac{-zK_{2/3}(w)}{\pi\sqrt{3}}$, where K_ν is the modified Bessel function and $w = 2z\sqrt{z}/3$.

For very large $|z|$, argument reduction will cause total loss of accuracy, and so no computation is performed. For slightly smaller $|z|$, the computation is performed but results are accurate to less than half of ***machine precision***. If $\mathrm{Re}(w)$ is too large, and the unscaled function is required, there is a risk of overflow and so no computation is performed. In all the above cases, a warning is given by the function.

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Amos D E 1986 Algorithm 644: A portable package for Bessel functions of a complex argument and non-negative order *ACM Trans. Math. Software* **12** 265–273

5 Parameters

5.1 Compulsory Input Parameters

1: **deriv – string**

Specifies whether the function or its derivative is required.

If **deriv** = 'F', $\mathrm{Ai}(z)$ is returned.

If **deriv** = 'D', $\mathrm{Ai}'(z)$ is returned.

Constraint: **deriv** = 'F' or 'D'.

2: **z – complex scalar**

The argument z of the function.

3: **scal – string**

The scaling option.

scal = 'U'

The result is returned unscaled.

scal = 'S'

The result is returned scaled by the factor $e^{2z\sqrt{z}/3}$.

Constraint: **scal** = 'U' or 'S'.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **ai – complex scalar**

The required function or derivative value.

2: **nz – int32 scalar**

Indicates whether or not **ai** is set to zero due to underflow. This can only occur when **scal** = 'U'.

If **nz** = 0, **ai** is not set to zero.

If **nz** = 1, **ai** is set to zero.

3: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

On entry, **deriv** $\neq$ 'F' or 'D'.
or **scal** $\neq$ 'U' or 'S'.

ifail = 2

No computation has been performed due to the likelihood of overflow, because Re(w) is too large, where $w = 2\mathbf{z}\sqrt{\mathbf{z}}/3$ – how large depends on **z** and the overflow threshold of the machine. This error exit can only occur when **scal** = 'U'.

ifail = 3

The computation has been performed, but the errors due to argument reduction in elementary functions make it likely that the result returned by s17dg is accurate to less than half of ***machine precision***. This error exit may occur if ABS(**z**) is greater than a machine-dependent threshold value.

ifail = 4

No computation has been performed because the errors due to argument reduction in elementary functions mean that all precision in the result returned by s17dg would be lost. This error exit may occur if ABS(**z**) is greater than a machine-dependent threshold value.

ifail = 5

No result is returned because the algorithm termination condition has not been met. This may occur because the parameters supplied to s17dg would have caused overflow or underflow.

7 Accuracy

All constants in s17dg are given to approximately 18 digits of precision. Calling the number of digits of precision in the floating-point arithmetic being used t, then clearly the maximum number of correct digits in the results obtained is limited by $p = \min(t, 18)$. Because of errors in argument reduction when computing elementary functions inside s17dg, the actual number of correct digits is limited, in general, by $p - s$, where $s \approx \max(1, |\log_{10}|z||)$ represents the number of digits lost due to the argument reduction. Thus the larger the value of $|z|$, the less the precision in the result.

Empirical tests with modest values of z, checking relations between Airy functions $\mathrm{Ai}(z)$, $\mathrm{Ai}'(z)$, $\mathrm{Bi}(z)$ and $\mathrm{Bi}'(z)$, have shown errors limited to the least significant 3 – 4 digits of precision.

8 Further Comments

Note that if the function is required to operate on a real argument only, then it may be much cheaper to call s17ag or s17aj.

9 Example

```
deriv = 'F';
z = complex(0.3, +0.4);
scal = 'U';
[ai, nz, ifail] = s17dg(deriv, z, scal)
```

```
ai =
   0.2716 - 0.1002i
nz =
           0
ifail =
           0
```